Wind Shear Terms in the Equations of Aircraft Motion

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Conventional analyses of aircraft motion in the atmosphere have neglected wind speed variability on the scales associated with many atmospheric phenomena such as thunderstorms, low level jets, etc These phenomena produce wind shears that have been determined as the probable cause in many recent commercial airline ac cidents This paper derives the 6 deg equations of motion for an aircraft incorporating the variable wind terms The equations are presented in several coordinate systems (i e , body coordinates, inertial coordinates, etc) The wind shear terms, including the temporal and spatial gradients of the wind, appear differently in the various coordinate systems; these terms are discussed. Also, the influence of wind shear on inputs to computing the aerodynamic coefficients, such as the effects of wind velocity vector rotation on relative angular rates of rotation and on the time rate of change of angles of attack and sideslip, are addressed

Nomenclature

$\bar{A}$	= aerodynamic force vector
C	= side force
D	= drag force
F	= frame of reference
g	= gravity
I	= moment and/or product of inertia
L	= lift force
$L\ \ell$	= rolling moment
M	= pitching moment
m	= mass
N	= yawing moment
O	= mass center of gravity
$p\ q\ r$	= rate of roll, pitch, and yaw, respectively
T	= thrust
u	= x component of aircraft velocity relative to the atmosphere
$\bar{V}_E$	= inertial velocity vector
V	= absolute value of relative velocity
v	= y component of aircraft velocity relative to the atmosphere
W	= wind velocity vector
w	= z component of aircraft velocity relative to the atmosphere
X	= x component of aerodynamic force
x	= distance along x axis
Y	= y component of aerodynamic force
y	= distance along y axis
Z	= z component of aerodynamic force
z	= distance along z axis
α	= angle of attack
β	= angle of sideslip
θ	= Euler angle (elevation)
ϕ	= Euler angle (bank)
ψ	= Euler angle (azimuth)
$\bar{\omega}$	= angular rotation vector

Superscripts

$(\dot{\ })$	= time derivative d( )/dt
$(\vec{\ })$	= vector quantity

Subscripts

E	= measured in the inertial coordinates
W	= wind field quantity
w	= measured in the wind coordinates
$x\ y\ z$	= measured in x y, and z directions, respectively

Introduction

LOW LEVEL wind shear is now recognized as a severe flight hazard [1] Investigations of at least 25 commercial airline accidents (the most recent involved the loss of a Boeing 727 during takeoff from New Orleans International Airport, July 1983) and at least five U.S Air Force (USAF) mishaps[2 3] have clearly proven that wind shear (a sudden change in either the speed or direction of the wind) can produce major adverse dynamic effects on aircraft performance These effects can cause the aircraft to deviate significantly from the pilot's intended flight path, producing impact with the ground or frightening near collisions Both the International Civil Aviation Organization (ICAO) and the Federal Aviation Administration (FAA) now recognize wind shear as a potential hazard to the safety of aircraft operations, especially in critical landing and takeoff phases of flight Prior to this recognition, the role wind shear played in aircraft accidents often may have gone undetected or been attributed to pilot error

It is not surprising that the possibility of losing control of an aircraft due to unusual and extreme wind variations had not been given adequate attention prior to 1975 when the Eastern 66 accident occurred Practically all textbooks (see, as examples, Refs. 4-9) and educational programs on aircraft flight dynamics place little, if any, emphasis on variable winds In general, constant or zero winds, both in the development of the governing equations and in the analyses of aircraft motion in the atmosphere, are assumed Etkin (Ref 4, pp 378 384) and McRuer et al. (Ref 9, pp 249 252) devote a few pages to the subject It is unlikely, however, that un dergraduate or even graduate programs address wind shear effects. Thus, the authors believe that a careful derivation of the aircraft equations of motion with wind shear terms in corporated should be documented

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Of course, numerous studies relative to the influence of individual gusts or random turbulence on flight performance of aircraft (see, for example, Refs 10 16) have been con ducted However, these studies address only aircraft per formance relative to changes in wind on time and spatial scales, which are small in comparison with the scales of severe wind shear associated with thunderstorms gust fronts, or microbursts.

In addition to the lack of attention by aerodynamicists to aircraft performance in wind shear on the spatial and tem poral scales of storms and microbursts, meteorologists only recently have began to measure these wind fields in the detail necessary to analyze their effect on the motion of aircraft [17 19] In 1976, Foy[19] presented some spatially two dimensional (three wind speed components) wind field models These models, developed from flight data recordings and ground-based tower measurements, are highly qualitative Recently the JAWS program has measured three dimensional time dependent wind fields[18] specifically for the purpose of in vestigating aircraft performance These wind fields are ex pressed in a volume grid array with values of the three wind speed components given at each grid point Volume in terpolation techniques are used to compute values for the wind speed components and their gradients at any given aircraft position.[20 21]

Frost et al.[20 21] address implementations of spatially dependent wind fields into computer analysis of aircraft motion, and Bowles[24] considers the implementation of them into flight simulators Incorporation of the wind field models into the governing equation of aircraft motion is not straightforward, however Several new terms appear in the equations (depending on the frame of reference chosen) which do not appear in the standard, more familiar formulation for the case of zero or constant winds Moreover, there is clearly a lack of agreement and understanding on the correct form of the terms and the magnitude of their effect

The purpose of this paper is to review the governing equations of motion and identify and describe additional terms which appear to be due to spatial and temporal variation in wind on the scale associated with wind shear caused accidents Assessing the magnitude of the effects due to accurately modeling these additional terms (i e , interpolation vs backward differencing, etc) is an on going project However, preliminary evaluation of these effects has been carried out and will be presented in a follow on paper

This paper also presents some considerations relative to evaluating aerodynamic coefficients for input to the equations; however research remains to be done to fully assess the effects of spatially varying winds on experimental and computed aerodynamic coefficients

The distinction between wind shear and turbulence (or gust gradients) is simply a matter of scale Work remains to be done to establish the magnitude of this scale The higher frequency wind fluctuations (i e turbulence) may increase the pilot's workload but not necessarily affect the general flight path of the aircraft Herein wind shear is defined in terms of those relatively long scale motions in the atmosphere which are consistent with the assumption that the wind acts uniformly or linearly over the aircraft at any given instant

General Equations of Unsteady Motion

Etkin[4] gives a complete development of the general equations of unsteady motion However the variation of wind velocity is not generally incorporated into the equations, i e , a zero or constant wind is assumed The form of the wind vector components in the governing equations depends on the coordinate system chosen Based on the assumption that the Earth is a stationary plane in inertial space, a coordinate system fixed at the Earth is defined as the inertial frame of reference, designated F_E The coordinate axes are designated x_E y_E and z_E The appropriate reference frame for com puting the motion of the aircraft subject to a ground wind is an atmosphere fixed reference frame, F_A, since the aerodynamic forces and moments depend on the velocity of the vehicle relative to the local atmosphere The velocity of the atmosphere relative to the Earth is the motion of F_A relative to F_E and is designated $\bar{W}$

Two other reference frames of interest are the air trajectory reference frame F_W (also called the wind-axis reference frame; this "wind" should not be confused with the at mospheric motion), and the body fixed reference frame F_B, or body axis reference frame The wind axis reference frame, F_W, has its origin fixed to the vehicle, usually at the mass center, and the axis is directed along the velocity vector of the vehicle relative to the atmosphere, $\bar{V}$ Thus,

$$\bar{V}=\bar{V}_E-\bar{W} \tag{1}$$

where $\bar{V}_E$ is the inertial velocity or the velocity of the vehicle relative to the fixed Earth The axis $O_w z_w$ lies in the plane of symmetry of the vehicle The frame F_W has angular velocity relative to the inertial frame F_E The components of the angular velocity are conventionally designated by P_w q_w and r_w

The body axes make up a reference frame fixed in a rigid body Bodies with articulated control surfaces and/or elastic motions for which the body cannot be taken as rigid are not considered in this equation development The origin of the body axes is usually the mass center of gravity, O The plane of symmetry is generally taken as Oxz, with z directed downward By convention, the components of angular velocity of the body axis frame of reference, F_B relative to F_E are designated p, q, and r The components along the body axis of the aircraft velocity relative to the atmosphere are denoted by u v, and w

Based on these assumptions, the force equations for six degree of-freedom motion become:

Force equations in inertial axes, F_E:

$$X_E=mx_E \qquad Y_E=my_E \qquad Z_E+mg=m\dot{z}_E \tag{2}$$

Force equations in wind axes, F_W:

$$T_{xw}-D-mg\sin\theta_w=m(\dot{V}+\dot{W}_{xw})+m(q_w W_{zw}-r_w W_{yw})$$

$$T_{yw}-C+mg\cos\theta_w\sin\phi_w=m\dot{W}_{yw}+m[r_w(V+W_{xw})-P_w W_{zw}]$$

$$T_{zw}-L+mg\cos\theta_w\cos\phi_w=m\dot{W}_{zw}+m[p_w W_{yw}-q_w\times(V+W_{xw})] \tag{3}$$

Force equations in body axes, F_B:

$$X-mg\sin\theta=m(u+\dot{W}_x)+m[q(w+W_z)+m[q(w+W_z)-r(v+W_y)]$$

$$Y+mg\cos\theta\sin\phi=m(\dot{v}+\dot{W}_y)+m[r(u+W_x)-p(w+W_z)]$$

$$Z+mg\cos\theta\cos\phi=m(\dot{w}+\dot{W}_z)+m[p(v+W_y)-q(u+W_x)] \tag{4}$$

Addressing the force equations first, it appears that the equations in the inertial axis have the simplest forms and do not explicitly depend on the wind vector or its gradient However, consider the expansions of the aerodynamic forces X_E Y_E and Z_E in terms of lift, drag, and side forces

Let the x_E axis of the inertial system be chosen as pointing north and L_{WE} be the coordinate transformation of the vector components in the inertial reference frame (x_E y_E z_E) into components in the wind frame of reference (x_W y_W z_W) This transform matrix, in terms of the Euler angles, has the form shown in Eq (5) The Euler angles are defined as those angles which rotate the Earth's fixed coordinates into coincidence with the relevant axis system (for example, F_W or F_B)

$$L_{WE}=\begin{bmatrix} \cos\theta_w\cos\psi_w & \cos\theta_w\sin\psi_w & -\sin\theta_w \\ \sin\phi_w\sin\theta_w\cos\psi_w - \cos\phi_w\sin\psi_w & \sin\phi_w\sin\theta_w\sin\psi_w + \cos\phi_w\cos\psi_w & \sin\phi_w\cos\theta_w \\ \cos\phi_w\sin\theta_w\cos\psi_w + \sin\phi_w\sin\psi_w & \cos\phi_w\sin\theta_w\sin\psi_w - \sin\phi_w\cos\psi_w & \cos\phi_w\cos\theta_w \end{bmatrix} \quad (5)$$

The transform to body coordinates, L_{BE}, is the same as Eq (5) with θ_w, ϕ_w and ψ_w replaced with θ, ϕ, and ψ, respec tively

The force component in the inertial axis thus becomes

$$\begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} = L_{BE}^T \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} - L_{WE}^T \begin{bmatrix} D \\ C \\ L \end{bmatrix} \quad (6)$$

where L_{WE}^T and L_{BE}^T are the transpose of L_{WE} and L_{BE}, respectively The wind vector is observed to enter not only through the aerodynamic coefficient relationships for D C and L (which are discussed later), but also through the Euler angles, which are functions of the wind vector The symbols T_x T_y, and T_z represent composite propulsive and control forces in body axes

To see this, consider the rotation ψ_w about the O_Ez_E axis, followed by a rotation θ_w about O_Ex_2 In terms of the inertial and wind vector

$$\psi_w = \tan^{-1}\left(\frac{\dot{y}_E - W_{yE}}{\dot{x}_E - W_{xE}}\right)$$

$$\theta_w = \tan^{-1}\left(\frac{\dot{z}_E - W_{zE}}{\sqrt{(\dot{x}_E - W_{xE})^2 + (\dot{y}_E - W_{yE})^2}}\right) \quad (7)$$

Note that ϕ_w is arbitrary However, for the case where y_w is defined perpendicular to the plane of $\chi_w\zeta_w$ ϕ_w represents a rotation about the x_w axis The value of ϕ_w is determined from the moment equation or kinematic relationships Despite the relatively innocuous appearance of wind effects on the force equation expressed in the inertial coordinate system, the wind vector obviously enters in a strong, inplicit manner

In general, use of the inertial coordinate system also requires transforming the thrust vector $\bar{T}$ from body coor dinates into the inertial areas The Euler angles ψ, θ, and ϕ must hen be evaluated For this reason, most computer programs and flight simulator systems use body coordinate axes.

Therefore, consider the wind vector and its gradient terms as they appear in the force equation expressed in the body coordinate frames of reference Transposing the wind vector terms in Eq (4) gives the rate of change of relative velocity as

$$(\dot{\bar{V}} + \bar{\omega} \times \bar{V}) = \bar{T}/m + \bar{A}/m + \bar{g} - (\dot{\bar{W}} + \bar{\omega} \times \bar{W}) \quad (8)$$

where $\bar{\omega}$ is the angular rotation of the body frame of reference relative to the Earth The wind vector terms $((\dot{\bar{W}} + \bar{\omega} \times \bar{W})$ appear as effective forces When their values become of the same order of magnitude as the thrust and weight to mass ratios, the wind shear is significant Frost[22] gives an order of magnitude analysis relative to these terms and suggests that wind shear is critical for a commercial type airliner when the two dimensional spatial shears are on the order of

$$\frac{\partial W_x}{\partial x} \simeq 0\ 02\text{s}^{-1}(1\ \text{knot}/100\ \text{ft})$$

$$\frac{\partial W_x}{\partial z} \simeq 0\ 32\text{s}^{-1}(19\ 5\ \text{knots}/100\ \text{ft})$$

$$\frac{\partial W_z}{\partial x} \simeq 0\ 13\text{s}^{-1}(7\ 7\ \text{knots}/100\ \text{ft})$$

$$\frac{\partial W_z}{\partial z} \simeq 2\ 61\text{s}^{-1}(159\ 1\ \text{knots}/100\ \text{ft})$$

These values are corroborated by flight simulator studies [24]

Consider now the evaluation of the wind velocity vector terms The components of the wind velocity vector are given most frequently in the Earth frame of reference The relationships between the Earth components and those in the aircraft body frame of reference F_B are given by

$$\begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} = L_{BE} \begin{bmatrix} W_{xE} \\ W_{yE} \\ W_{zE} \end{bmatrix} \quad (9)$$

$$W_x = W_{xE}\cos\theta\cos\psi + W_{yE}\cos\theta\sin\psi - W_{zE}\sin\theta$$

$$W_y = W_{xE}(\sin\phi\sin\theta\cos\psi - \cos\theta\sin\psi) + W_{yE}(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + W_{zE}\sin\phi\cos\theta$$

$$W_z = W_{xE}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) + W_{yE}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) + W_{zE}\cos\phi\cos\theta \quad (10)$$

The velocity vector components in F_W are the same with the Euler angles (ϕ, θ, ψ) replaced by (ϕ_w θ_w, ψ_w)

The total derivatives of the wind vector components are

$$\dot{W}_{xE} = \left(\frac{\partial W_x}{\partial x}\right)_E \dot{x}_E + \left(\frac{\partial W_x}{\partial y}\right)_E \dot{y}_E + \left(\frac{\partial W_x}{\partial z}\right)_E \dot{z}_E + \left(\frac{\partial W_x}{\partial t}\right)_E$$

$$\dot{W}_{yE} = \left(\frac{\partial W_y}{\partial x}\right)_E \dot{x}_E + \left(\frac{\partial W_y}{\partial y}\right)_E \dot{y}_E + \left(\frac{\partial W_y}{\partial z}\right)_E \dot{z}_E + \left(\frac{\partial W_y}{\partial t}\right)_E$$

$$\dot{W}_{zE} = \left(\frac{\partial W_z}{\partial x}\right)_E \dot{x}_E + \left(\frac{\partial W_z}{\partial y}\right)_E \dot{y}_E + \left(\frac{\partial W_z}{\partial z}\right)_E \dot{z}_E + \left(\frac{\partial W_z}{\partial t}\right)_E \quad (11)$$

The derivatives can be computed by either of two procedures: compute the components of $\dot{W}$ in the inertial (Earth's) frame of reference and then transform the resulting vector into the body or wind frame of reference Note, however, that the transform of Eq (11) is tedious because the transform of the deviative $\dot{W}$ requires

$$\begin{bmatrix} \dot{W}_x \\ \dot{W}_y \\ \dot{W}_z \end{bmatrix} = L_{BE} \begin{bmatrix} \dot{W}_{xE} \\ \dot{W}_{yE} \\ \dot{W}_{zE} \end{bmatrix} + \dot{L}_{BE} \begin{bmatrix} W_{xE} \\ W_{yE} \\ W_{zE} \end{bmatrix} \tag{12}$$

where

$$\dot{L}_{BE} = \begin{bmatrix} -(\sin\theta\cos\psi)\dot{\theta} & -(\sin\theta\sin\psi)\dot{\theta} & -(\cos\theta)\dot{\theta} \\ -(\cos\theta\sin\psi)\dot{\psi} & +(\cos\theta\cos\psi)\dot{\psi} & \\ (\cos\phi\sin\theta\cos\psi & (\cos\phi\sin\theta\sin\psi & (\cos\phi\cos\theta)\dot{\phi} \\ +\sin\phi\sin\psi)\dot{\phi} & -\sin\phi\cos\psi)\dot{\phi} & -(\sin\phi\sin\theta)\dot{\theta} \\ +(\sin\phi\cos\theta\cos\psi)\dot{\theta} & +(\sin\phi\cos\theta\sin\psi)\dot{\theta} & \\ +(-\sin\phi\sin\theta\sin\psi & +(\sin\phi\sin\theta\cos\psi & \\ -\cos\phi\cos\psi)\dot{\psi} & -\cos\phi\sin\psi)\dot{\psi} & \\ (\sin\phi\sin\theta\cos\psi & (-\sin\phi\sin\theta\sin\psi & -(\sin\phi\cos\theta)\dot{\phi} \\ +\cos\phi\sin\psi)\dot{\phi} & -\cos\phi\cos\psi)\dot{\phi} & -(\cos\phi\sin\theta)\dot{\theta} \\ +(\cos\phi\cos\theta\cos\psi)\dot{\theta} & +(\cos\phi\cos\theta\sin\psi)\dot{\theta} & \\ +(-\cos\phi\sin\theta\sin\psi & +(\cos\phi\sin\theta\cos\psi & \\ +\sin\phi\cos\psi)\dot{\psi} & +\sin\phi\sin\psi)\dot{\psi} & \end{bmatrix} \tag{13}$$

Again, $\dot{L}_{WE}$ is the same with θ, ϕ, and ψ replaced by θ_w ϕ_w and ψ_w respectively

Alternately, one can take the derivatives in the body or wind frames of reference arriving at

$$\dot{W}_x = \left(\frac{\partial W_x}{\partial x}\right)_B (u+W_x) + \left(\frac{\partial W_x}{\partial y}\right)_B (v+W_y) + \left(\frac{\partial W_x}{\partial z}\right)_B \times (w+W_z) + \left(\frac{\partial W_x}{\partial t}\right)_B$$

$$\dot{W}_y = \left(\frac{\partial W_y}{\partial x}\right)_B (u+W_x) + \left(\frac{\partial W_y}{\partial y}\right)_B (v+W_y) + \left(\frac{\partial W_y}{\partial x}\right)_B \times (w+W_z) + \left(\frac{\partial W_y}{\partial t}\right)_B$$

$$\dot{W}_z = \left(\frac{\partial W_y}{\partial x}\right)_B (u+W_x) + \left(\frac{\partial W_z}{\partial y}\right)_B (v+W_y) + \left(\frac{\partial W_z}{\partial x}\right)_B \times (w+W_z) + \left(\frac{\partial W_z}{\partial t}\right)_B \tag{14}$$

To implement Eq (14), the wind speed spatial derivatives, which nearly always are given in the Earth frame of reference, must be transformed to the body or wind frames of reference Since $\vec{\nabla}\vec{W}$ is a dyad, or second order tensor, there are nine terms

$$\begin{bmatrix} \frac{\partial W_x}{\partial x} & \frac{\partial W_y}{\partial x} & \frac{\partial W_z}{\partial x} \\ \frac{\partial W_x}{\partial y} & \frac{\partial W_y}{\partial y} & \frac{\partial W_z}{\partial y} \\ \frac{\partial W_x}{\partial z} & \frac{\partial W_y}{\partial z} & \frac{\partial W_z}{\partial z} \end{bmatrix} \tag{15}$$

The dyad transforms as

$$\vec{\nabla}_B \vec{W} = L_{BE}^T \vec{\nabla}_E \vec{W} L_{BE} \tag{16}$$

The expansion of Eq (16) requires 54 multiplications and 36 additions for every time step of the solution analysis However, Eq (12) requires only 18 multiplications and 12 additions But on the other hand, use of Eq (12) requires evaluation of $\dot{L}_{BE}$, which takes several additional multiplications and additions Thus, the computational expedience of one technique over the other is mainly a matter of preference

As with the inertial coordinate system, the aerodynamic forces must be transformed from the wind frame of reference to the body frame of reference The transformation matrix which rotates the wind coordinates into body coordinates is

$$L_{BW} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \tag{17}$$

Values of α and β are computed from

$$\tan\alpha = w/u \qquad \sin\beta = v/V \tag{18}$$

Hence

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} - L_{BW} \begin{bmatrix} D \\ C \\ L \end{bmatrix} \tag{19}$$

The rates of angular rotation p q and r are generally computed in the body coordinate system, regardless of the coordinate system in which the force equation is expressed The reason for this is that the moments of inertia are relatively constant in the body axis The moment equation based on the assumption of a rigid aircraft with symmetry in the Oxz plane is

$$L = I_x\dot{p} - I_{zx}(\dot{r}+pq) - (I_y - I_z)qr$$
$$M = I_y\dot{q} - I_{zx}(r^2 - p^2) - (I_z - I_x)rp$$
$$N = I_z\dot{r} - I_{zx}(\dot{p} - qr) - (I_x - I_y)pq \tag{20}$$

Equation (20) can be solved for $\dot{p}$ $\dot{q}$ and $\dot{r}$ and the dif ferential equation solved by standard techniques for p q and r

Since the angular rotations are inertial values, they are not dependent on the wind vector The aerodynamic moments, L, M and N however, do depend on the wind vector and its gradient, as will be addressed later

Values of ϕ, θ, and ψ are computed from kinematic relationships

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \tag{21}$$

The same relationships hold for ϕ_w θ_w and ψ_w Again, neither the wind vector nor its gradient enters explicitly enters the computation, for ϕ, θ, and ψ

Finally the position of the aircraft relative to inertial space is computed from

$$\dot{x}_E = u\cos\theta\cos\psi + v(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + w(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) + W_{xE}$$
$$\dot{y}_E = u\cos\theta\sin\psi + v(\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi) + w(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) + W_{yE}$$
$$\dot{z}_E = -u\sin\theta + v\sin\psi\cos\theta + w\cos\psi\cos\theta + W_{zE} \tag{22}$$

Aerodynamic Forces

The aerodynamic forces and moments which act on aircraft are in general, complicated functions of shape and motion The values of these forces are known only approximately How these are influenced by variable winds is even less well understood In this section we do not pretend to address aerodynamic forces in depth but only to point out a few areas which need consideration when dealing with spatially and temporally varying winds

In general, expressions for aerodynamic forces and moments are expressed by the relationships

$$D = C_D\rho V^2 S/2 \qquad C = C_C\rho V^2 S/2 \qquad L = C_L\rho V^2 S/2 \tag{23a}$$

and

$$\ell = C_\ell\rho V^2 Sb/2 \qquad M = C_M\rho V^2 S\bar{c}/2 \qquad N = C_N\rho V^2 Sb/2 \tag{23b}$$

where $\bar{c}$ = wing mean chord, b = wing span,

$$V^2 = (\dot{x}_E - W_{xE})^2 + (\dot{y}_E - W_{yE})^2 + (\dot{z}_E - W_{zE})^2 \tag{24a}$$

in inertial coordinates, and

$$V^2 = u^2 + w^2 + w^2 \tag{24b}$$

in body coordinates The coefficients C_D C_C, C_L C_ℓ C_M and C_N are expanded in terms of a linear series of variables upon which they are dependent In general, these expansions contain time dependent derivatives of α and β (i e , $\dot{\alpha}$ and $\dot{\beta}$) Also, the coefficients are linearly related to the angular rates of rotation When variable wind effects are considered, care must be exercised to assure that these wind effects are computed relative to the air mass and not the inertial system

Consider first $\dot{\alpha}$ and $\dot{\beta}$ Differentiating $\tan\alpha = w/u$ and $\sin\beta = v/V$ with respect to time and carrying out the algebra gives

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2}$$
$$\dot{\beta} = \frac{\dot{v}(u^2 + w^2) - v(u\dot{u} + w\dot{w})}{V^2\sqrt{u^2 + w^2}} \tag{25}$$

Note that the wind speed gradients enter Eq (25) through $\dot{u}$ $\dot{v}$, and $\dot{w}$, which are given by [see Eq (4)]

$$\dot{u} = \frac{X}{m} - g\sin\theta - \dot{W}_x - [q(w + W_z) - r(v + W_y)]$$
$$\dot{v} = \frac{Y}{m} + g\cos\theta\sin\phi - \dot{w}_y - [r(u + W_x) - p(w + W_z)]$$
$$\dot{w} = \frac{Z}{m} + g\cos\theta\cos\phi - \dot{W}_z - [p(v + W_y) - q(u + W_x)] \tag{26}$$

Thus, wind shear has a direct effect on these variables and, hence, the attendant aircraft responses

Claims often are made that by using the inertial coordinate system, calculation of the wind speed gradients can be avoided Such is not the case, however, since in terms of inertial coordinates

$$\tan\alpha = [u_E(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) + v_E(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) + w_E\cos\phi\cos\theta]/(u_E\cos\theta\cos\psi + v_E\cos\theta\sin\psi - w_E\sin\theta) \tag{27}$$

where

$$u_E = (\dot{x}_E - W_{xE}) \qquad v_E = (\dot{y}_E - W_{yE}) \qquad w_E = (\dot{z}_E - W_{zE}) \tag{28}$$

Differentiating the above with respect to time results in a much more complex expression than Eq (25) and contains several wind speed gradients A similar but equally complex ex pression exists for β and $\dot{\beta}$ when expressed in the inertial coordinate system

Expressions for the aerodynamic coefficients generally contain linear relationships in terms of the angular rotations p, q and r Equation (20) normally is used to solve the dynamic equations of motion for p q and r The values obtained are thus inertial values However, the stability derivatives multiplied by p q and r in the linear expansion of the aerodynamic coefficients are based on values of angular rotation relative to the atmosphere Thus, care must be taken to assure that $(\bar{\omega} - \bar{\omega}_w)$ is used in evaluating the aerodynamic coefficients The rotation vector $\bar{\omega}_w$ is the rotation of the wind field relative to the inertial frame of reference Figure 1 illustrates the concept for a two dimensional configuration Recall that the tensor gradient of a vector field can be split into symmetric and antisymmetric parts [25] The symmetric part measures the instantaneous rate of strain while the antisymmetric part measures the instantaneous rate of rotation at the point considered Physically, this rotation can be viewed as an air mass of a rigid solid and is brought about by fluid stresses whose curl does not vanish Thus, $\bar{\omega}_w$ can be expressed as

$$\bar{\omega}_W = \tfrac{1}{2}\,\bar{\nabla}\times\bar{W} \tag{29}$$

In the inertial coordinates

$$\bar{\omega}_W = \frac{1}{2}\left(\frac{\partial W_z}{\partial y}-\frac{\partial W_y}{\partial z}\right)_E\bar{i}+\frac{1}{2}\left(\frac{\partial W_x}{\partial z}-\frac{\partial W_z}{\partial x}\right)_E\bar{j}+\frac{1}{2}\times\left(\frac{\partial W_y}{\partial x}-\frac{\partial W_x}{\partial y}\right)_E\bar{k} \tag{30}$$

Recall that for body coordinates, $\bar{\omega}_W$ must be transformed with L_{BE}, hence,

$$\begin{bmatrix} p_{\rm rel}\\ q_{\rm rel}\\ r_{\rm rel}\end{bmatrix}=\begin{bmatrix} p\\ q\\ r\end{bmatrix}-L_{BE}\begin{bmatrix}\left(\frac{\partial W_z}{\partial y}-\frac{\partial W_y}{\partial z}\right)_E\\ \left(\frac{\partial W_x}{\partial z}-\frac{\partial W_z}{\partial x}\right)_E\\ \left(\frac{\partial W_y}{\partial x}-\frac{\partial W_x}{\partial y}\right)_E\end{bmatrix} \tag{31}$$

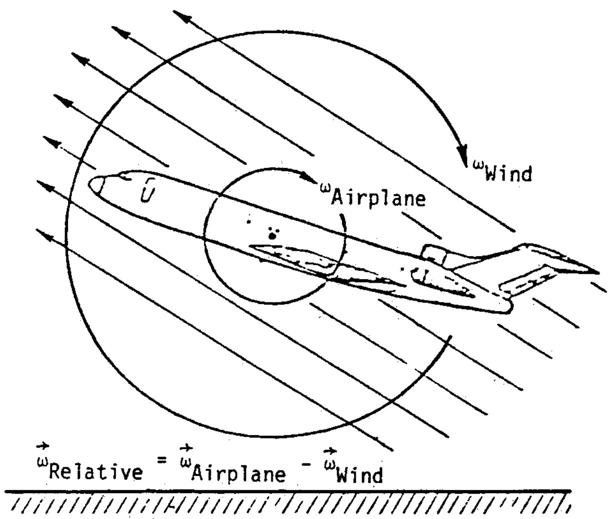

Fig 1 Schematic illustration of wind field rotation

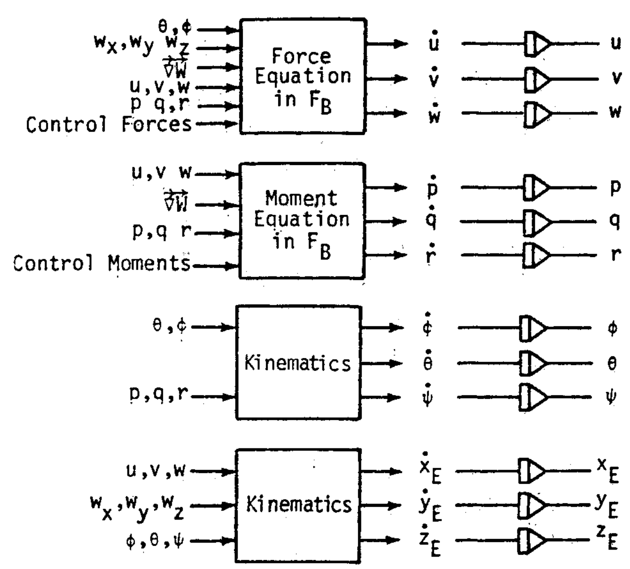

Fig 2 Block diagram of equations for vehicle with plane of sym metry flat Earth approximation; body coordinates

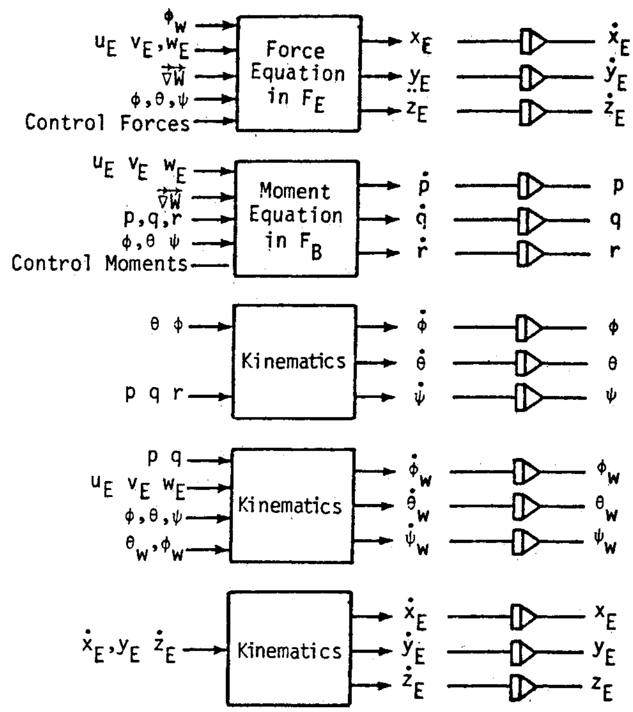

Fig 3 Block diagram of equations for vehicle with plane of sym metry flat Earth approximation; inertial coordinates

The aerodynamic roll, pitch, and yaw rates are thus strongly dependent on the spatial gradient of wind speed which is wind shear

Computational Flowcharts

Figures 2 and 3 outline the recommended computational procedure for evaluating the aircraft equations of motion (The flowchart concept borrows liberally from Etkin [4]) Figure 2 is for a body coordinate system, whereas Fig 3 is for an inertial coordinate system The wind coordinate system, which can be handled similarly is not discussed in detail in this paper

Conclusions

A consistent derivation of the governing equations of motion for aircraft performance in spatially and temporally varying wind fields results in additional terms appearing in the equations These terms are functions of the wind vector and its derivatives In body coordinates, the terms act as effective forces in the force equation In inertial coordinates they do not appear explicitly in the force equations However, the wind vector and its derivatives implicitly influence the transformation of the lift, drag, and crosswind forces into components in the inertial axes. Moreover, they appear directly and in a more complex form in evaluation of the aerodynamic coefficients Thus, although it is often argued that the inertial system is the simplest to use in computing aircraft motion in variable winds, the results of this paper clearly suggest that the body coordinate system is still preferred

To update existing computer programs or flight simulator computation packages requires slight but straightforward corrections to existing computational schemes utilizing zero mean or constant wind models Variable wind effects can be added directly to the forces acting on the aircraft in body coordinates All other modifications appear in the calculations of the derivatives of angles of attack and sideslip and in evaluating the relative rates of angular rotation These modifications can influence the calculation of aerodynamic coefficients directly but do not appear explicitly in the governing equations of motion

Preliminary evaluation of the effect of the wind vector and its derivatives show that the terms are significant It is thus recommended that computer codes and flight simulator programs be updated to incorporate the additional wind shear terms.

The accuracy of and optimum computational procedure for evaluation of the wind speed gradients are being investigated Backward differencing of the time derivative terms is ob viously much more simple than interpolation for all spatial derivatives Preliminary results utilizing the JAWS data sets however, show differences in predicted touchdown patterns of as much as 100% for backward differencing vs interpolation Research is continuing in this area and a workshop to fully explore this and other topics relative to implementation of wind shear models is being planned at NASA Langley Research Center

Until this work is completed, the magnitude of the ad ditional wind field terms given in this paper is not fully known; however, it is concluded that these terms must appear in the governing equations of aircraft motion if consistent and reliable computer analyses and flight simulator studies of the impact of wind shear on aircraft performance are to be achieved

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References

[1] *Low Altitude Wind Shear and Its Hazard to Aviation* National Academy Press Washington, D C , 1983

[2] U S Air Force Wind Shear Conference Travis Air Force Base Calif May 1978

[3] Tinsley H G Coons, F G and Wood L W , Possible Near Term Solutions to the Wind Shear Hazard *Preprints: Air Tran sportation Meeting*, Society of Automotive Engineers Inc Warrendale Pa 1978.

[4] Etkin B *Dynamics of Atmospheric Flight* John Wiley & Sons Inc , New York 1972 pp. 121 195

[5] Perkins C D and Hage R E. *Airplane Performance Stability and Control* John Wiley & Sons, Inc New York 1963

[6] Houghton, E L and Brock A E *Aerodynamics for Engineering Students* St Martin s Press, New York, 1972 pp 135 219

[7] Babister A W *Aircraft Dynamic Stability and Response* Pergamon Press New York 1980 pp 98 100

[8] Roskam, J *Flight Dynamics of Rigid and Elastic Airplanes Part 1*, printed by the author The University of Kansas Lawrence Kan 1973 pp. 11 22

[9] McRuer D Ashkenas, I and Grahm D *Aircraft Dynamics and Automatic Control* Princeton University Press Princeton N J 1973, pp 203 220.

[10] Houbolt, J C Steiner R , and Pratt, K G ,Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Response,' NASA TR R 199 1964

[11] Peloubet R P and Haller R L , Application of a Power Spectral Gust Design Procedure to Bomber Aircraft AFFDL TR 66 35, June 1966

[12] Wang S T and Frost W Atmospheric Turbulence SImulation Techniques with Application to Flight Analysis NASA CR 3309 1980

[13] Flight in Turbulence *AGARD Conference Proceedings* AGARD CP 140 1973

[14] Reeves P M Joppa, R G and Ganzer V M A Non Gaussian Model of Continuous Atmospheric Turbulence for Use in Aircraft Design NASA CR 2639 1976

[15] Frost W , Camp D W and Wang S T Thunderstorm WInd Shear Modeling for Aircraft Hazard Definition ' *WMO Technical Conference on Aviation Meteorology* Geneva Swit zerland Nov 1979, pp 5 85

[16] Goff C Measuring Weather for Aviation Safety in the 1980's ' *Proceedings: Fourth Annual Workshop on Meteorology and Environmental Inputs to Aviation Systems* NASA CP 2139/FAA-RD 80 67 1980 pp 82 108

[17] Frost, W Camp D W and Wang, S T , Wind Shear Modeling for Aircraft Hazard Definition " U S Dept of Tran sportation, Washington D C , FAA RD 78 3 1978

[18] McCarthy, J JAWS Interim Report for Second Year s Ef fort " JAWS Project Boulder Colo NCAR Rpt 02 84 Sept 1983

[19] Foy W H Airborne Aids for Coping with Low Level Wind Shear, ' U S Dept of Transportation Washington D C FAA RD 79 117 1979

[20] Frost W , Chang H. P McCarthy J and Elmore K Aircraft Performance in a JAWS Microburst ' *Preprints: 21st Conference on Radar Meteorology,* Sept 19 23 1983, Edmonton Alberta Canada American Meteorological Society Boston Mass pp 630 637

[21] Frost W Chang H P , Elmore K L and McCarthy J ' Simulated Flight Through JAWS Wind Shear *Journal of Aircraft*, Vol 21 Oct 1984, pp 797 802

[22] Frost W. Flight in Low Level Wind Shear NASA CR 3678 1983

[23] Frost, W and Crosby W A Investigations of Simulated Aircraft Flight Through Thunderstorm Outflows NASA CR 3053 1978.

[24] Bowles R L Presentation at JAWS Training Simulator Workshop Boulder Colo Oct 1983

[25] Karamcheti K , *Principles of Ideal Fluid Aerodynamics* John Wiley & Sons Inc New York 1966 pp 231 235